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$$=L[1+\frac{W\sin\varphi}{2M}].\quad \therefore \frac{WL\sin\varphi}{2M} \text{ is the elongation.}$$

Also solved by the Proposer.

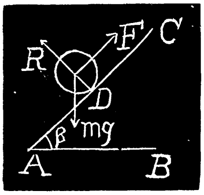
201. Proposed by G. B. M. ZERR, Ph. D., Parsons, W. Va.

ABC is an inclined plane, perfectly rough, length $AC=l$. The time for a sphere to roll down when AB is base is to the time for a cylinder to roll down when BC is base as m is to n. Find AB and BC.

Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Let O be the center of any rolling body of mass m; then the three forces that will act on it are its weight mg vertically downward, the resistance R on the inclined plane AC, and the friction F acting up the plane.

Denoting CD by s, we have therefore $m(\partial^2 s/dt^2)=mg\sin\beta-F$; and if we denote by θ the angular velocity, reducing the mass to the center O, we have $m\rho^2(\partial^2\theta/\partial t^2)=Fr$, r being equal to DO, and ρ radius of gyration. Since there is no sliding, the plane being perfectly rough, we have $s=r\theta$. Eliminating F, we have $\partial^2 s/dt^2=[r^2/(r^2+\rho^2)]g\sin\beta$, and integrating



$$s=\frac{r^2}{r^2+\rho^2}g\sin\beta\, t^2.$$

For the sphere $\rho^2=\frac{2}{5}r^2$, and for the cylinder $\rho^2=\frac{1}{2}r^2$; therefore, by the condition of the problem, for the sphere

$$l=\frac{r^2}{r^2+\frac{2}{5}r^2}g\,.\frac{BC}{l}t^2,\text{ whence } t^2=\frac{14l^2}{5g.\overline{BC}},$$

and for the cylinder, $l=\frac{r^2}{r^2+\frac{1}{2}r^2}g\,.\frac{AB}{l}t^2$, whence $t^2=\frac{3l^2}{g.\overline{AB}}$; $\therefore \frac{14}{5\overline{BC}}:\frac{3}{\overline{AB}}=m^2$

$:n^2$, and combining this with $\overline{AB}^2+\overline{CB}^2=l^2$, we get

$$AB=\frac{14n^2l}{\sqrt{(225m^4+196n^4)}},\quad CB=\frac{15m^2l}{\sqrt{(225m^4+196n^4)}}.$$

Also solved by J. Edward Sanders, and the Proposer.

AVERAGE AND PROBABILITY.

185. Proposed by R. D. CARMICHAEL, Anniston, Ala.

If a line l is divided into n parts by $n-1$ points taken at random on it, what is the mean value of the pth power of one of the parts taken at random?

Solution by HENRY HEATON, Belfield, N. D.

By making the line the circumference of a circle it will be seen that each part has exactly the same chance, so that the average of one part is the same as that of any other. Hence it will be sufficient to find the average length of any one part.

Let $x_1, x_2, \ldots, x_{n-1}$ be the distance of the different points from one extremity of the line. Then the average length of $(l-x_{n-1})^p$ is

$$A=\frac{\int_0^l\int_0^{x_{n-1}}\ldots\int_0^{x_2}(l-x_{n-1})^p dx_{n-1}dx_{n-2}\ldots dx_1}{\int_0^l\int_0^{x_{n-1}}\ldots\int_0^{x_2}dx_{n-1}dx_{n-2}\ldots dx_1}$$

$$=\frac{\int_0^l(l-x_{n-1})^p x_{n-1}^{n-2}dx_{n-1}}{\int_0^l x_{n-1}^{n-2}dx_{n-1}}=(n-1)l^p B[(p+1),\ (n-1)]$$

$$=\frac{(n-1)l^p\,[\Gamma(p+1),\ \Gamma(n-1)]}{\Gamma(p+n)}=\frac{l^p\,[\Gamma(p+1),\ \Gamma(n)]}{\Gamma(p+n)}$$

or, the average length of $x_1{}^p$ is

$$A=\frac{\int_0^l\int_0^{x_{n-1}}\ldots\int_0^{x_2}dx_{n-1}\,dx_{n-2}\ldots x_1{}^p dx_1}{\int_0^l\int_0^{x_{n-1}}\ldots\int_0^{x_2}dx_{n-1}\,dx_{n-2}\ldots dx_1}$$

$$=\frac{l^p(n-1)!\,p!}{(p+n-1)!}=\frac{l^p[\Gamma(n),\ \Gamma(p+1)]}{\Gamma(p+n)}.$$

Also solved by G. B. M. Zerr.

186. Proposed by G. B. M. ZERR, Ph. D., Parsons, W. Va.

An urn contains $n=100$ balls; $a=25$ balls are stamped, at random, with the letter A; $b=30$ balls are stamped, at random, with the letter B; $c=40$ balls are stamped, at random, with the letter C; $d=50$ balls are stamped, at random, with the letter D. One ball is drawn at random; find the chance it has on it no letter, the letter A, or B, or C, or D, or the letters AB, AC, AD, BC, BD, CD, ABC, ABD, ACD, BCD, or $ABCD$.

Solution by J. E. SANDERS, Reinersville, O., and the PROPOSER.

$(a/n)=\frac{25}{100}=\frac{1}{4}$, $(1-a/n)=\frac{3}{4}$; $(b/n)=\frac{30}{100}=\frac{3}{10}$, $(1-b/n)=\frac{7}{10}$; $(c/n)=\frac{40}{100}=\frac{2}{5}$, $(1-c/n)=\frac{3}{5}$, $(d/n)=(1-d/n)=\frac{50}{100}=\frac{1}{2}$.

∴ The required chances are in order: